Quiz 1.3: Sample Answers

1. For what value of c is

$$f(x) = \begin{cases} cx + 2 & \text{if } x \leq 4; \\ cx^2 - 4 & \text{if } 4 < x \end{cases}$$

cts on $(-\infty, \infty)$?

It is cts everywhere except possibly at $c = 4$. To get it cts there, we must set the left and right limits equal. So we need:

$$\begin{aligned} \lim_{x \to 4^-} f(x) &= \lim_{x \to 4^+} f(x) \\ \lim_{x \to 4^-} cx + 2 &= \lim_{x \to 4^+} cx^2 - 4 \\ c(4) + 2 &= c(4)^2 - 4 \\ 4c + 2 &= 16c - 4 \\ 12c &= 6 \\ c &= \frac{1}{2} \end{aligned}$$

2. For what value of c is

$$f(x) = \begin{cases} x^2 - c^2 & \text{if } x \leq -5; \\ cx + 125/4 & \text{if } -5 < x \end{cases}$$

cts on $(-\infty, \infty)$?

It is cts everywhere except possibly at $c = -5$. To get it cts there, we must set the left and right limits equal. So we need:

$$\begin{aligned} \lim_{x \to (-5)^-} f(x) &= \lim_{x \to (-5)^+} f(x) \\ \lim_{x \to (-5)^-} x^2 - c^2 &= \lim_{x \to (-5)^+} cx + 125/4 \\ (-5)^2 - c^2 &= c(-5) + 125/4 \\ 25 - c^2 &= -5c + 125/4 \\ c^2 + 5c + 125/4 - 100/4 &= 0 \\ c^2 + 5c + 25/4 &= 0 \\ (c + 5/2)(c + 5/2) &= 0 \\ c &= 5/2 \end{aligned}$$

3. Evaluate
$$\lim_{x\to\infty} \frac{-x^3+4x^2-x+4}{-3x^3+x^2+4x+2}$$
As usual for limits as x goes to ∞, we first divide everything by the highest power of x, in this case x^3:
$$= \lim_{x\to\infty} \frac{-x^3/x^3+4x^2/x^3-x/x^3+4/x^3}{-3x^3/x^3+x^2/x^3+4x/x^3+2/x^3}$$
$$= \lim_{x\to\infty} \frac{-1+4/x-1/x^2+4/x^3}{-3+1/x+4/x^2+2/x^3}$$
Then, as x goes to ∞, any term with a negative power of x goes to 0:
$$= \frac{-1+0-0+0}{-3+0+0+0}$$
$$= 1/3$$

4. Evaluate
$$\lim_{x\to\infty} \sqrt{4x^2+6x}-2x$$
As with most limits involving a root, we must first multiply and divide by the conjugate:
$$= \lim_{x\to\infty} \sqrt{4x^2+6x}-2x\left(\frac{\sqrt{4x^2+6x}+2x}{\sqrt{4x^2+6x}+2x}\right)$$
$$= \lim_{x\to\infty} \frac{(4x^2+6x)-4x^2}{\sqrt{4x^2+6x}+2x}$$
$$= \lim_{x\to\infty} \frac{6x}{\sqrt{4x^2+6x}+2x}$$
We then divide by the highest power of x. In this case, since the x^2 is under a square root sign, the highest power is x.
$$= \lim_{x\to\infty} \frac{6x/x}{(1/x)\sqrt{4x^2+6x}+2x/x}$$
As the bottom $1/x$ goes into the square root sign, it must become squared to remain the same:
$$= \lim_{x\to\infty} \frac{6}{\sqrt{4x^2/x^2+6x/x^2}+2}$$

$$= \lim_{x\to\infty} \frac{6}{\sqrt{4 + 6/x} + 2}$$

Finally, we can take the limit as x goes to ∞:

$$= \frac{6}{\sqrt{4+0}+2}$$

$$= \frac{6}{2+2}$$

$$= 3/2$$